Quantum mechanics II, Chapter 5 : Measurement and decoherence

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You can't sit with us.

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Problem 1 : Environment induced decoherence

1. Consider a composite system that is prepared in the initial state $|\psi\rangle = \sum_j c_j |E_j\rangle_A \otimes |\phi\rangle_B$ and evolves under a Hamiltonian $H_{AB} = \sum_j |E_j\rangle\langle E_j|_A \otimes H_B^{(j)}$ for time t where $\{|E_j\rangle\}$ forms a basis of system A.

   Find an expression for the reduced states $\rho_A(t)$ and $\rho_B(t)$ of systems A and B as a function of time.
2. Under what circumstances do A and B remain pure for all times ?
3. Under what circumstances does $\rho_A(t)$ become approximately diagonal in the basis $\{|E_j\rangle\}$?
4. Why do you think $H_{AB} = \sum_j |E_j\rangle\langle E_j|_A \otimes H_B^{(j)}$ is sometimes described as quantifying the "*measurement limit* of system-environment interactions" ? (Bonus question (discuss with a partner) : how is this related to the measurement problem ?)

Problem 2 : Decoherence and dephasing of a single qubit

1. Consider applying a random R_z rotation, i.e. $e^{-i\sigma_z\vartheta/2}$ for $\vartheta \in [-\pi, \pi]$ to a generic initial pure qubit state $|\psi\rangle = \sin(\theta)|0\rangle + \cos(\theta)e^{-i\phi}|1\rangle$. What is the resulting mixed state on average ? Sketch this on the Bloch sphere.

   Disclaimer : What does "average mixed state" even mean ? In this case, it means that we take the average over all states with respect to this σ_z rotation. How do we take the average ? We just compute

$$\rho_{\mathrm{av}} = \frac{1}{2\pi} \int_{-\pi}^{\pi} R_z(\vartheta) \,|\psi\rangle \,\langle\psi|\, R_z^\dagger(\vartheta) d\vartheta \tag{1}$$

2. Consider now applying a random R_z rotation and then a random R_x rotation. What is the resulting mixed state on average ? And what if you now apply all three (a random R_z, R_x and R_y) ? Sketch this on the Bloch sphere.
3. What if instead you apply a random R_z rotation with probability p and do nothing with probability $1-p$? And what if you apply a random R_z rotation then a random R_x rotation with probability p, and do nothing with probability $1-p$? Sketch this on the Bloch sphere.

4. Suppose you now instead throw away your initial state and prepare the maximally mixed state $\mathbb{I}/2$ with probability p and do nothing with probability $1-p$? Sketch this on the Bloch sphere.
5. What do you conclude from all this ?